# Ch. 5 Relationships within Triangles

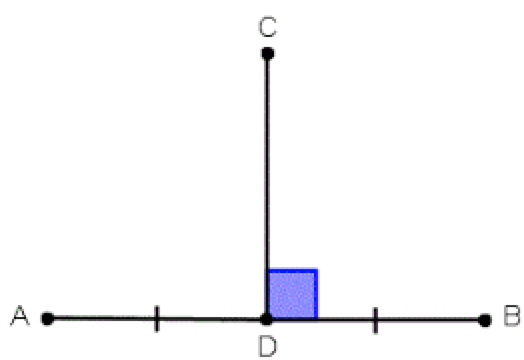
# 5.1 Midsegment Theorem and Coordinate Proof

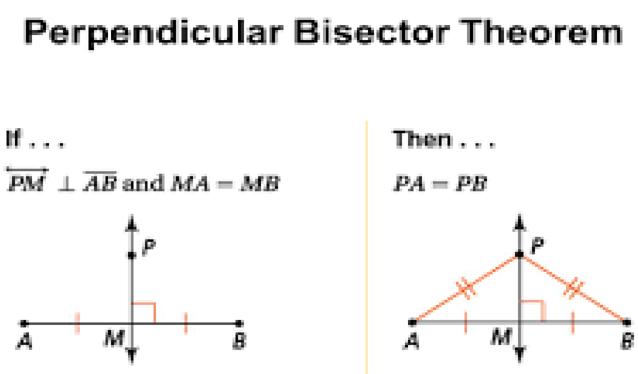
- **Midsegment**: segment that connects midpoints.
  - One triangle has 3 midsegments
  - Midsegment parallel to side
  - Midsegment = $\frac{1}{2}$ side

- **Coordinate Proof**: place a figure on a graph
  - Use origin as one of vertices
- **Formulas** to Remember:
  - Distance: $d = \sqrt{(x_2 - x_1)^2} + (y_2 - y_1)^2$
  - Midpoint: $\left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$
  - Slope: = $\frac{y_2 - y_1}{x_2 - x_1}$

# 5.2 Use Perpendicular Bisectors

- **Perpendicular Bisector:**
  - Can be a ray, line, plane, segment
  - Is perpendicular at the midpoint
  - All points on the perpendicular bisector are **equidistant** from endpoints of segment



- **Concurrency**: when 3 or more lines intersect ant the same point (point of concurrency)

- **Theorem**: If the segments of perpendicular bisectors meet at point, then distance from point to vertex is equal.
- **Circumcenter**: point of concurrency of 3 perpendicular bisectors.
  - P circumcenter
  - P same distance to all 3 vertices
  - P center of circle outside triangle that touches all 3 vertices

# 5.3 Use Angle Bisectors of Triangle

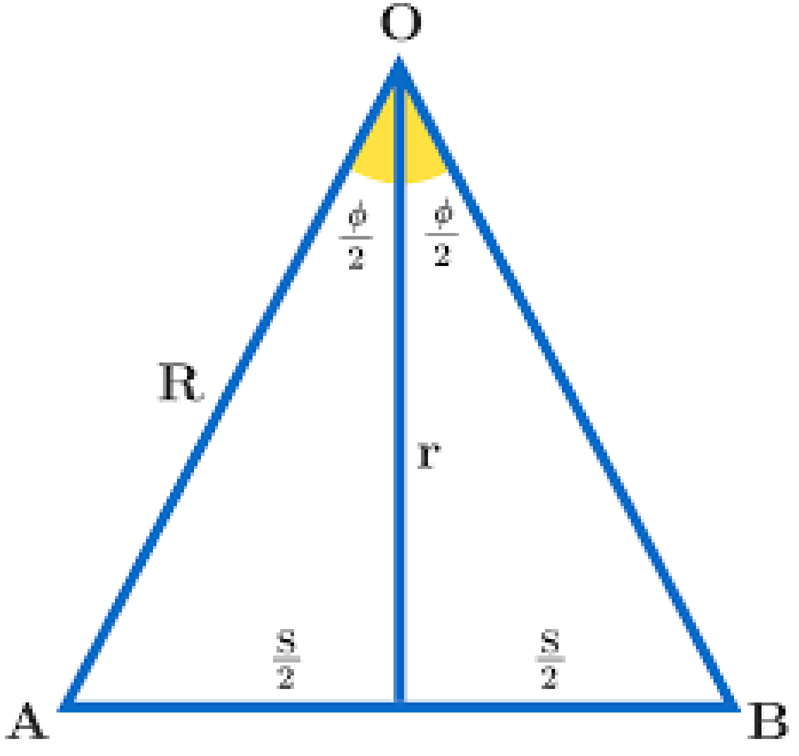
- 1. Midsegment
- 2. Perpendicular bisector of segment
- 3. **Angle Bisector**: ray that divides an angle into 2 $\cong$ angles
  - Distance: perpendicular segment
- **Theorem**: If angle bisector and perpendicular, then distance to bisector equal
- Converse true

- **Theorem: Concurrency of Angle Bisector $\Delta$**
  - The point of concurrency of the angle bisectors is equal distance to the triangle
  - P incenter:  center of circle inside triangle

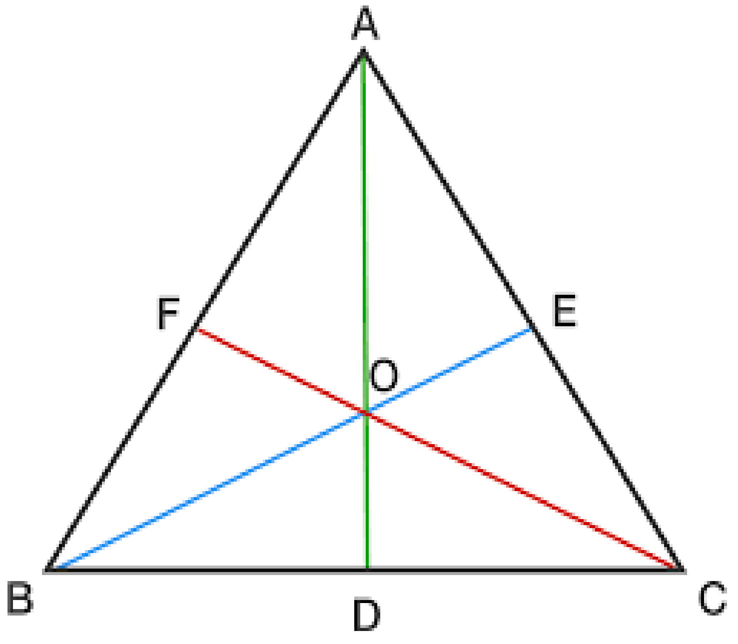
# 5.4 Medians and Altitudes

- **Median**: balance point in triangle
  - Segment from vertex to midpoint
  - 3 medians concurrent: centroid
  - vertex to centroid = $\frac{2}{3}$ segment
- **Altitude**: Height of triangle
  - Segment from vertex to side, perpendicular
  - In triangle, on triangle, or outside triangle

- **Isosceles Triangle**:
  - From Vertex:
  - Perpendicular Bisector
  - Angle Bisector
  - Median
  - Altitude
  - ALL EQUAL



- **Equilateral Triangle**:
  - From ANY angle
  - Perpendicular Bisector
  - Angle Bisector
  - Median
  - Altitude
  - ALL EQUAL



# Points in a Triangle:

- **Incenter**:  when angle bisectors  meet.
  - Distance to sides equal
- **Circumcenter**: when perpendicular bisectors meet.
  - Distance to vertices equal
- **Centroid**:  when medians meet
  - Distance = $\frac{2}{3}$ segment
- **Orthocenter**: when altitudes meet
  - Can be in triangle, on triangle, or out of triangle

# Summary of Parts

- 1. **Midsegment**: midpoint to midpoint
- 2. **Perpendicular Bisector**:  thru midpoint
- 3. **Angle Bisector**: cuts angle
- 4. **Median**: from angle to midpoint
- 5. **Altitude**:  height from angle to ground

# 5.5 Inequalities in a Triangle

- **In a Triangle**:
  - The longest side is opposite the largest angle
  - The shortest side is opposite the smallest angle

- **Theorem**:
  - If AB > BC, Then *m*<C > *m*<A
- Converse:
  - If *m*<C > *m*<A, Then AB > BC

- Can’t always make a triangle with 3 sticks
- The sticks must be a certain length to work

- **Theorem**: The sum of any 2 sides of a triangle must be > third side

# 5.6 Inequalities in 2 Triangles and Indirect Proof

- **Hinge Theorem**:
  - If 2 triangles have 2 sides congruent and the included angle gets bigger, then the third side gets bigger. Involves 2 triangles

- **Indirect Proof: Proof of Contradiction**
  - Starts by assuming the opposite of concluded proof