Chapter 3: Powers, Roots, and Radicals

3.1 nth Roots and Rational Exponents

- Exponent Form: $a^{m/n}$ Ex: $2^{3/4}$

- Root Form: $\sqrt[n]{a^m}$ Ex: $\sqrt[4]{2^3}$

3.2 Properties of Rational Exponents

- **Exponent Form:** $a^{m/n}$

- When Simplifying: Use properties of Exponents
  - **Bases** must be the same

  EX: $x^{1/3} \cdot x^{1/3} = x^{2/3}$ Add exponents

  $\left(x^{1/3}\right)^2 = x^{2/3}$ Multiply exponents

  $\dfrac{x^{2/3}}{x^{1/3}} = x^{1/3}$ Subtract exponents

  $\sqrt[3]{x^6} = x^2$ Divide exponents

- **Root Form:** $\sqrt[n]{a^m}$

- Can Multiply and Divide with same **index**
- Can add and subtract as like terms
- To Simplify: no perfect roots and
  - no roots in denominator
- Ex: $\sqrt[3]{2} \cdot \sqrt[3]{7} = \sqrt[3]{14}$

- Ex: $\dfrac{\sqrt[3]{10}}{\sqrt[3]{2}} = \sqrt[3]{\dfrac{10}{2}}$

- Ex: $5\sqrt[3]{2} + 7\sqrt[3]{2} = 12\sqrt[3]{2}$

- EX: $\sqrt[3]{16} = \sqrt[3]{8 \ x \ 2} = 2\sqrt[3]{2}$

3.3 Power Functions and Function Operations

- Functions:
  - Can Add, Subtract, Multiply, or Divide 2 functions

  $f(x) = x + 1$ $g(x) = 2x + 3$

  $f(x) + g(x)$

  $f(x) - g(x)$

  $f(x) \cdot g(x)$

  $f(x)/g(x)$

- Compositions: f(g(x)) and g(f(x))

- Putting one function into another

3.4 Inverse Functions

- To find **Inverse**:
  - switch x and y, solve for y   Ex: y = 2x − 1

    x = 2y − 1

    y = $\frac{x+1}{2}$

- To determine if 2 equations are inverses:
  - f(g(x)) = x AND g(f(x)) = x

- **Graph**:
  - Equation and Inverse are reflected about y = x.
  - Sometimes Inverse of function NOT function
    - **Vertical Line Test**: determines if an equation is a function
    - **Horizontal Line Test**: determines if its **Inverse** is a function

3.5 Graphing Square Root and Cube Root Functions

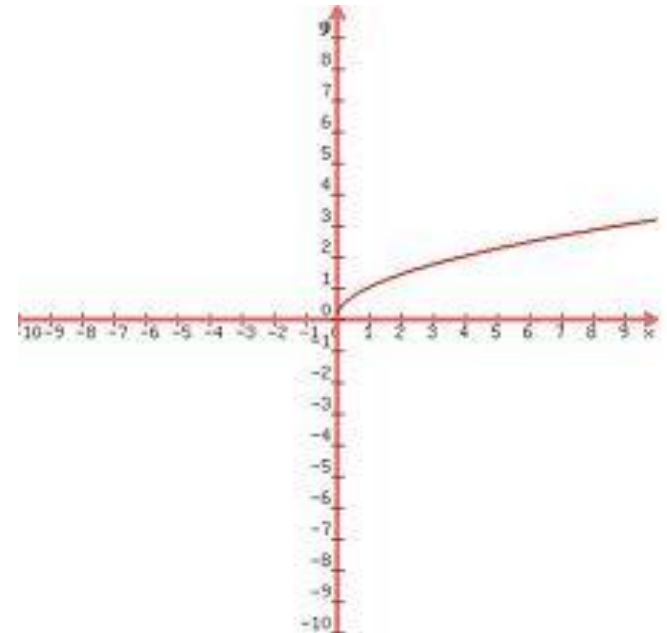- **Square Root Functions**:
  - y = $\sqrt{x}$
  - Domain: x $\geq$ 0
  - Range: y $\geq$ 0
  - Shifts: y = a $\sqrt{x - h}$ + k
  - *h* units horizontally
  - *k* units vertically
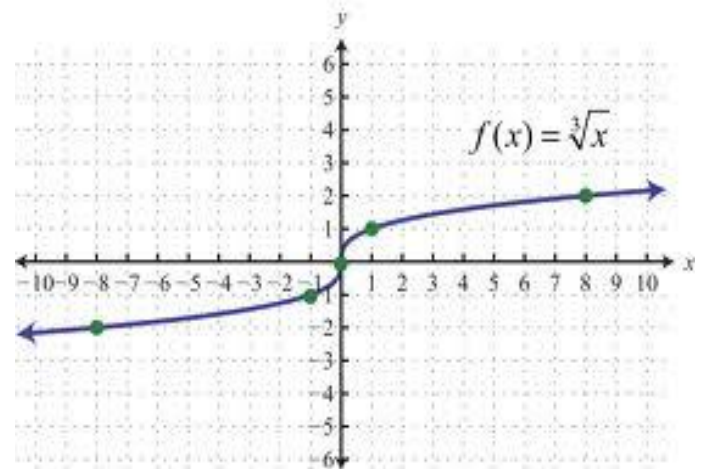
- **Cube Root Functions**:
- $y = \sqrt[3]{x}$
- Domain: all real
- Range: all real
- Shifts: $y = a\sqrt[3]{x - h} + k$
  - h units horizontally
  - k units vertically

$f(x) = \sqrt[3]{x}$

3.6 Solving Radical Equations

- Eliminate radicals and rational exponents
- Raise each side of the equation to the same power
- Check for extra solutions that don't work.