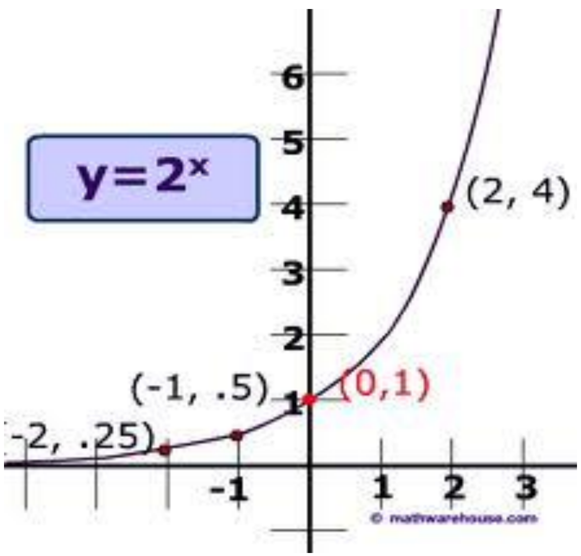
Exponential and Logarithmic Functions

Chapter 4

4.1 Exponential Growth

- Exponential Function: f(x) = a·b^x



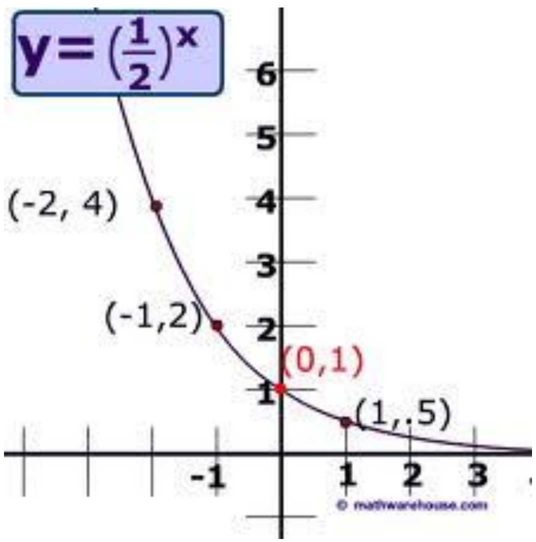
- Asymptote: x-axis; y = 0
- Domain: all real
- Range: y≥0
- Growth: a>0, b>1
- Shift: y = $ab^{x-h} + k$

- Increase at a fixed percent: y = $a(1 + r)^t$
  - Growth factor: 1 + r

- Compound Interest: A = $P(1 + \frac{r}{n})^{nt}$
- P Initial Principal
- r rate(decimal)
- n times per year
- t years

4.2 Exponential Decay

- Exponential Decay: f(x) = a·b^x



Decay: $a > 0$, $0 < b < 1$

Asymptote: $y = 0$

Domain: all real

Range: $y > 0$

Shift: $y = ab^{x-h} + k$

- Quantity decreases by fixed percent:
  - (Loses value each year)

    - y = $a(1-r)^t$

    - Decay factor: 1 – r

4.3 The number *e*

- In growth function: As n→∞, $(1+\frac{1}{n})^n$ = e
- *e = 2.71828*
- Natural base *e*
- Use as a base (rules apply)
- f(x) = ae^{rx} *f(x)* = e^x or $f(x) = e^{-x}$
  - Exponential Growth: a>0, r>0
  - Exponential Decay: a>0, r<0

- Continuously Compounded Interest:

$$A = Pe^{rt}$$

4.4 Logarithmic Functions

- Invented to “**solve for exponent**”
- A logarithm is the exponent
  - The answer to the log is an exponent
- Logarithmic Form: $\log_b y$ = x

- Exponential Form: b^x = *y*

- 2 special logarithms:

  - $\log_b 1 = 0$ because $b^0 = 1$
  - $\log_b b = 1$ because $b^1 = b$

- Common logarithm: base 10 = log x
- Natural logarithm: base *e* = *ln x*

- **Inverses**: exponential function and log function
  - "undo each other"
- $\log_b b^x$ = x $\qquad\qquad$ $b^{\log_b x}$ = x

- **To find inverse**: switch x and y
  - Solve for y (switch form)

- **To Graph**: y = $\log_b(x - h)$ + k

  Vertical Asymptote: line x = h

  Domain: x > h

  Range: all real numbers

  b > 1 up to right; 0<b<1 down to right

  Use 2 special logarithms

4.5 Properties of Logarithms

- Product: $\log_b uv = \log_b u + \log_b v$
- Quotient: $\log_b \frac{u}{v} = \log_b u - \log_b v$
- Power: $\log_b u^n$ = n $\log_b u$
- Change of Base: $\log_c u = \frac{\log u}{\log c}$ (allows calculator)

4.6 Solving Exponential and Logarithmic Equations

- **To Solve Exponential Equations:**

- **With the Same base**
  - Cancel the base. Set exponents = to each other.

- Ex: $2^{x+3} = 2^{5x-1}$

  $x + 3 = 5x - 1$

- **With NOT the Same Base**
  - **Make the bases the same**

- Ex: $2^{x+1} = 8^{x-3}$

  $2^{x+1} = (2^3)^{x-3}$

  x + 1 = 3x – 9

- **To Solve Logarithm Equations:**

- **With the same Logs**
  - **Cancel the logs**
- Ex: $\log(3x+1) = \log(x + 2)$

$$3x + 1 = x + 2$$

- **With log on one side**:
  - Change to exponential form

- Ex: $\log_2 x + 1$ = 3

  2^3 = x + 1