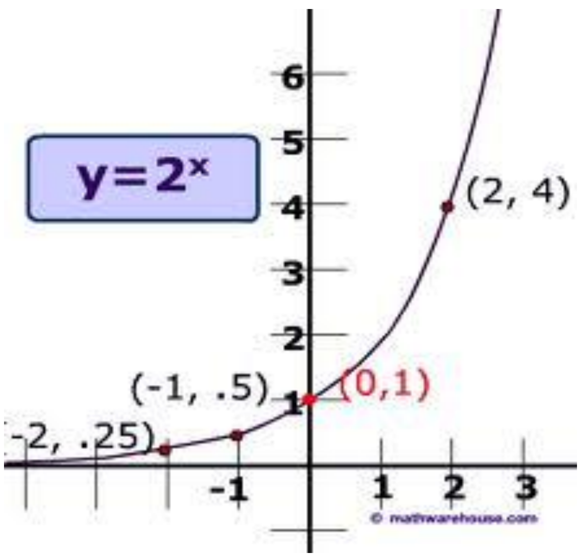
Exponential and Logarithmic Functions

Chapter 4

4.1 Exponential Growth

- Exponential Function: f(x) = a·b^x



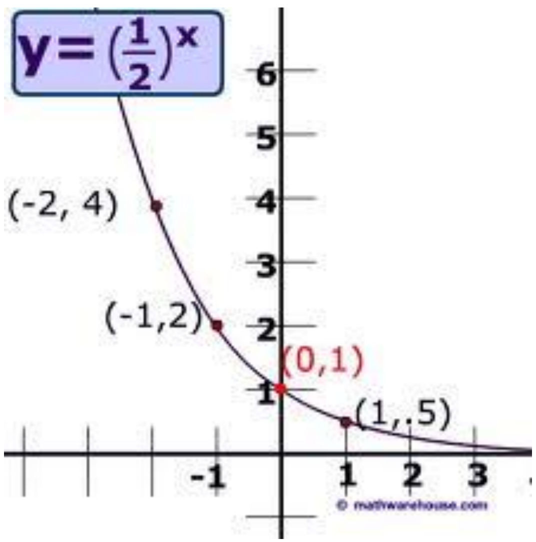
- Asymptote: x-axis; y = 0
- Domain: all real
- Range: y$\geq$0
- Growth: a>0, b>1
- Shift: y = ab^{x-h} + k

- Increase at a fixed percent: y = $a(1+r)^t$
  - Growth factor: 1 + r

- Compound Interest: A = $P(1+\frac{r}{n})^{nt}$
- P Initial Principal
- r rate(decimal)
- n times per year
- t years

4.2 Exponential Decay

- Exponential Decay: f(x) = a·b^x



Decay: a > 0, 0 < b < 1

Asymptote: y = 0

Domain: all real

Range: y > 0

Shift: y = ab^{x-h} + k

- Quantity decreases by fixed percent:
  - (Loses value each year)

    - y = $a(1-r)^t$

    - Decay factor: 1 – r

4.3 The number *e*

- In growth function: As n→∞, $(1+\frac{1}{n})^n$ = *e*
- *e = 2.71828*
- Natural base *e*
- Use as a base (rules apply)
- f(x) = ae^{rx} *f(x)* = e^x or $f(x) = e^{-x}$
  - Exponential Growth: a>0, r>0
  - Exponential Decay: a>0, r<0

- Continuously Compounded Interest:

$$A = Pe^{rt}$$

4.4 Logarithmic Functions

- Invented to "**solve for exponent**"
- A logarithm is the exponent
  - The answer to the log is an exponent
- Logarithmic Form: $\log_b y$ = x

- Exponential Form: b^x = *y*

- 2 special logarithms:

  - $\log_b 1 = 0$ because $b^0 = 1$
  - $\log_b b = 1$ because $b^1 = b$

- Common logarithm: base 10 = log x
- Natural logarithm: base *e* = *ln x*

- **Inverses**: exponential function and log function
  - "undo each other"
- $\log_b b^x$ = x $\quad\quad\quad\quad$ $b^{\log_b x}$ = x

- **To find inverse**: switch x and y
  - Solve for y (switch form)

- **To Graph**: y = $\log_b(x - h)$ + k

Vertical Asymptote: line x = h

Domain: x > h

Range: all real numbers

b > 1 up to right; $0<b<1$ down to right

Use 2 special logarithms

4.5 Properties of Logarithms

- Product: $\log_b uv = \log_b u + \log_b v$
- Quotient: $\log_b \frac{u}{v} = \log_b u - \log_b v$
- Power: $\log_b u^n$ = n $\log_b u$
- Change of Base: $\log_c u = \frac{\log u}{\log c}$ (allows calculator)

4.6 Solving Exponential and Logarithmic Equations

- To Solve **Exponential Functions**: $y = b^x$
  - Use same bases

    OR
  - Take log of each side
    - Use Properties if needed
  - Check solutions

- To Solve **Logarithm Functions**: $y = \log_b x$
- Cancel out same log

OR

- Change to exponential form
- Check extra solutions

4.7 Writing Exponential and Power Equations

- To write **exponential function**: $y = a \cdot b^x$
  - Given two points:
    - write 2 equations and use substitution
    - Solve for *a* and *b*

- To write **power function**: $y = a \cdot x^b$
  - Given two points:
    - write 2 equations and use substitution
    - Solve for *a* and *b*